Design-oriented stability criteria of a v^2 control compensated with inductor current of a Boost Converter for Shipboard Power Systems

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***Abstract*—This paper deals with remote control of power converters for marine applications where a boost converter is controlled with a v^2 control compensated with inductor current. This control has a fast inner loop where the control is able to react independently from the reference signal and a slow outer loop that changes the reference signal to regulate tightly the output voltage. This way, the control is decoupled in two parts where the fast inner loop is on-board of the ship and the slow outer loop is remotely controlled on-shore. As robustness of the system is required, the stability of the inner control has to be analyzed. This paper proposes a design-oriented stability criterion for the boost converter controlled with v^2 control compensated with inductor current so that designers can select the parameters of the control and know how to design the power stage in order to increase the stability region of the control.**

I. Introduction and Motivation

Power electronics converters are key elements in next-generation all electric marine systems and ships [1, 2], due to their various applications that allow for increased levels of control of nodal and flow characteristics. The operation of this type of systems is comprised of multiple independently operated converters that work together with electromechanical equipment. With the increase in the automation of system functions within this type of power electronic-based shipboard-power systems, the dependence on appropriate control strategies to maintain system performance at an adequate level is becoming more apparent.

In remote control of power converters, all or a part of the control system is placed in a distanced location. As the information of the sensors of the power stage and the reference signal of the control system need to be transmitted through a communication network, there is an inherent delay that can affect the stability of the system [3]. In this scenario, v^2-type control systems are very suitable [4, 5]. These controls have two control loops: a fast inner one that can react quickly under perturbations and a slow outer one that regulates the output voltage tightly. Figure 1 shows a scheme of the proposed control system for marine applications.

The fast inner control is on-board of the ship next to the power stage. The output voltage is converted to a digital signal and sent to the slow outer control. When designing such control schemes, nonlinear phenomena associated with power electronics converter systems must be taken into account[6, 7]. Since v^2-type controls are prone to sub-harmonic oscillations, their stability over the range of operation needs to be analyzed.

This work focuses on the development of a design-oriented stability criterion for a boost converter

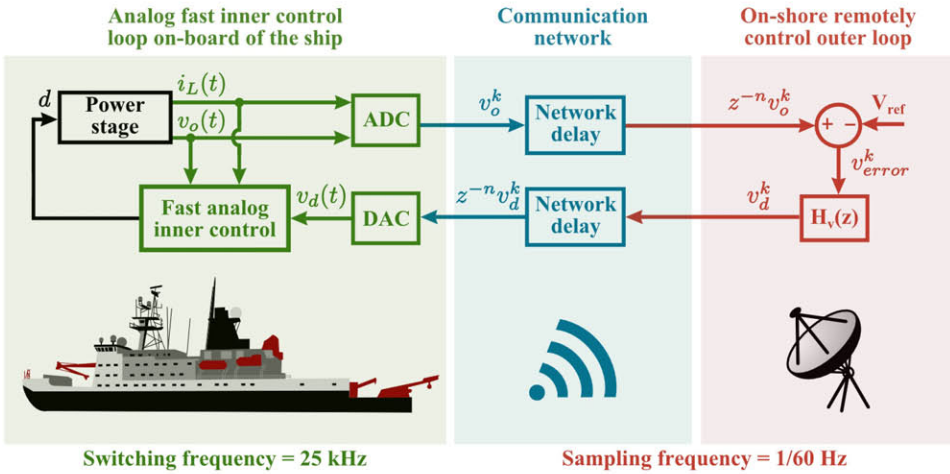


Fig. 1: Conceptual scheme of whole converter system.

with v^2 control compensated with inductor current [8] that will allow appropriate selection of control parameters to ensure stable operation.

II. Modeling and stability analysis

Figure 2 shows the scheme of a boost converter with v^2 control compensated with inductor current modulated with constant frequency. The control uses the output voltage weighted with a gain K_v, the inductor current weighted with a gain K_i and a compensating ramp V_{ramp} as the modulator signal for the reference voltage v_d. The outer control of fig.1 sets the reference voltage for the inner control, $v_d(t)$, which is updated at a rate of $\frac{1}{60}$ Hz. Notice that, for the fast inner loop, the reference voltage $v_d(t)$ is almost a constant because its bandwidth is much larger than the sampling frequency of the outer loop.

The approach for the stability analysis is based on [5], used for a v^2 control with constant on-time modulation. It consists on finding out the simplified discrete model of the power stage and evaluating its Jacobian matrix. This Jacobian matrix is known in periodic systems as the Monodromy matrix, which relates the perturbation at the beginning of the period to the perturbation at the end of the period. If all eigenvalues are inside the unit circle, then the perturbation does not grow over the period and, consequently, the system is stable.

Figure 3 shows the scheme of the boost converter for the on-state (fig.3a) and the off-state (fig.3b).

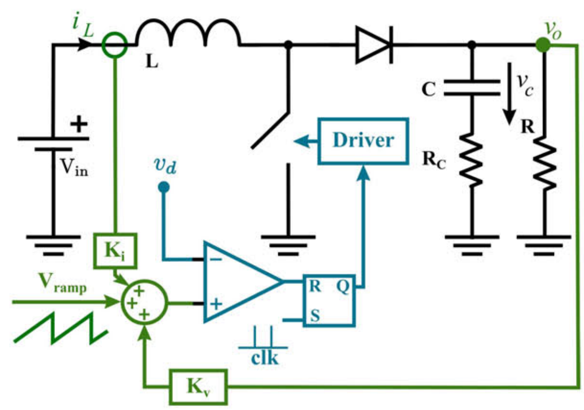


Fig. 2: Scheme of a boost converter with v^2 control compensated with inductor current modulated with constant frequency.

Without considering parasitic elements such as on-resistances of the switches, ESR of the inductor or ESR of the output capacitor, and assuming that the ripple of the output voltage does not affect the slope of the inductor current, it can be found that the value of the inductor current at the end of the period, $i_{L,k+1}$, as a function of the value of the inductor at the beginning of the period, $i_{L,k}$ is:

$$i_{L,k+1} = i_{L,k} + m_1 T_{on} - m_2 (T - T_{on}) \qquad (1)$$

where m_1 is the on-time slope of the inductor current, m_2 is the off-time slope of the inductor current, T is the period and T_{on} is the on-time.

Similarly, the value of the capacitor voltage at the end of the period, $v_{c,k+1}$, as a function of the value

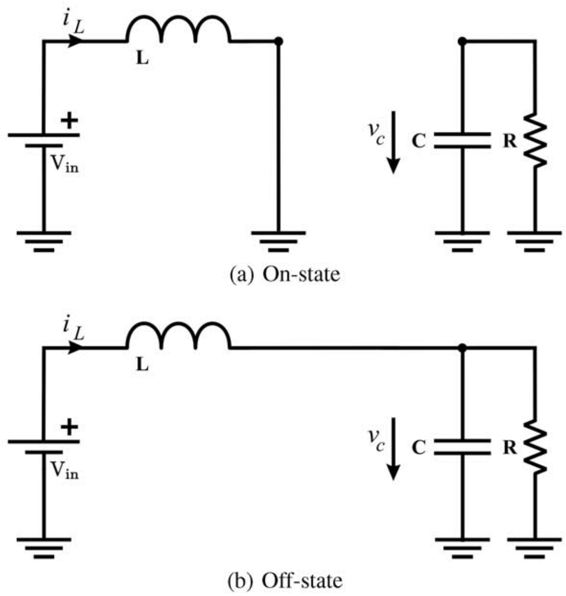


Fig. 3: On-state and Off-state of a Boost converter without considering the ESR of the output capacitor.

of the capacitor voltage at the beginning of the period, $v_{c,k}$ is:

$$v_{c,k+1} = v_{c,k} - \frac{1}{C} i_o T_{on} + \frac{i_{L,k} - i_o}{C}(T - T_{on}) + \frac{m_1 T_{on}}{C}(T - T_{on}) - \frac{m_2}{2C}(T - T_{on})^2 \quad (2)$$

As the converter is modulated with constant frequency, the control changes the length of the on-time to achieve the desired duty cycle. The condition to switch from on-state to off-time can be defined with a hypersurface h that depends on the output voltage ,$v_o(t)$, the inductor current, $i_L(t)$ and the time t:

$$h(i_L(t),\ v_c(t),\ t) = v_d - K_i i_L(t) - K_v v_o(t) - m_c t \quad (3)$$

where m_c is the slope of the compensating ramp.

At $t = T_{on}$, the converter changes from on-state to off-state and $h(i_L(T_{on}),\ v_c(T_{on}),\ T_{on}) = 0$. By relating $h(i_L(T_{on}),\ v_c(T_{on}),\ T_{on})$ to $i_{L,k}$, $v_{c,k}$ and T_{on}, and accounting now for R_c, the ESR of the output capacitor, it is found that:

$$h(i_{L,k},\ v_{c,k},\ T_{on}) = v_d - \left(v_{c,k} - \frac{1}{C} i_o T_{on} - i_o R_c\right) K_v - (i_{L,k} + m_1 T_{on}) K_i - m_c T_{on} = 0 \quad (4)$$

Equations (1), (2) and (4) form the implicit discrete model of the converter.

Now, by calculating the partial derivatives of the state variables applying the chain rule, it can be found that the Jacobian matrix, Φ, is:

$$\Phi = \begin{pmatrix} \frac{\partial i_{L,k+1}}{\partial i_{L,k}} & \frac{\partial i_{L,k+1}}{\partial v_{c,k}} \\ \frac{\partial v_{c,k+1}}{\partial i_{L,k}} & \frac{\partial i_{L,k+1}}{\partial v_{c,k}} \end{pmatrix} \quad (5)$$

where

$$\frac{\partial i_{L,k+1}}{\partial i_{L,k}} = \frac{-Cm_c + K_v i_o + Cm_2 K_i}{-Cm_c + K_v i_o - Cm_1 K_i} \quad (6)$$

$$\frac{\partial i_{L,k+1}}{\partial v_{c,k}} = \frac{(m_1 + m_2) C K_v}{-Cm_c + K_v i_o - Cm_1 K_i} \quad (7)$$

$$\frac{\partial v_{c,k+1}}{\partial i_{L,k}} = \frac{-K_i i_{L,k} - m_c T_{off} + K_v i_o T_{off}/C + m_1 T_{off}(K_i - 1)}{-Cm_c + K_v i_o - Cm_1 K_i} \quad (8)$$

$$\frac{\partial v_{c,k+1}}{\partial v_{c,k}} = \frac{-Cm_c + K_v(i_o - i_{L,k}) - Cm_1 K_i + m_1 T_{off} K_v}{-Cm_c + K_v i_o - Cm_1 K_i} \quad (9)$$

By analyzing the eigenvalues of the Jacobian matrix (5), it is concluded that converter is stable for the following conditions:

$$\text{Condition 1)} \quad K_i > 0 \quad (10)$$

$$\text{Condition 2)} \quad m_c > \frac{v_o}{L}(d - 0.5) K_i \quad (11)$$

$$\text{Condition 3)} \quad K_v > -\frac{v_{ramp}}{v_o} \frac{1}{2d + 1} \quad \text{and} \quad (12)$$

$$K_v < K_i \frac{C}{L}(1 - d)\frac{v_o}{i_o} + \frac{v_{ramp}}{v_o} \frac{1}{2d + 1} \quad (13)$$

where v_{ramp} is the amplitude of the compensating ramp and d is the duty cycle $d = T_{on}/T$. Notice that design criterion does not depend on the ESR of the output capacitor and it depends on the output current.

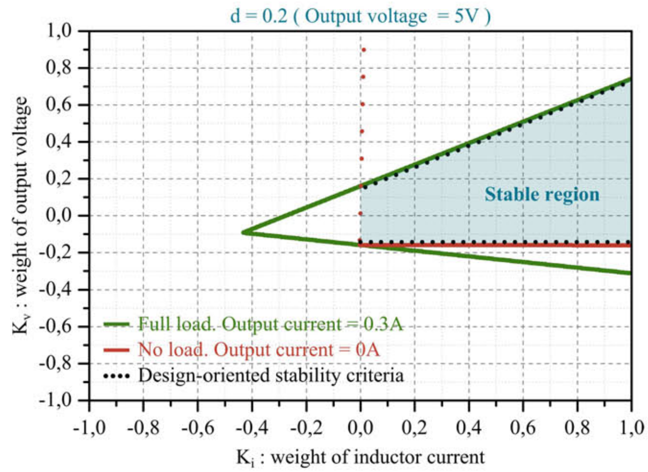
Notice that, in order to ensure stability, the gain of the inductor current always has to be greater than zero which means that, for the boost converter, it is needed to add current information to v^2-type controls modulated with constant frequency. On the other hand, the gain of the output voltage could be zero so, from the point of view of stability, there is no need to add the output voltage in the fast inner loop.

III. Comparison with accurate stability analysis

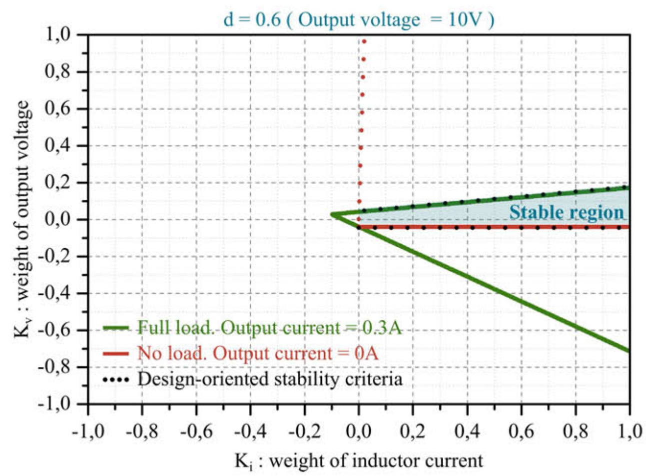
This section compares the proposed design-oriented criteria with a very accurate stability analysis based on numerical resolution [9]. The parameters of the power stage are: $f_{sw} = 25kHz$, $V_{in} = 4V$, $i_o = 0A - 3A$, $L = 5mH$, $C = 220\mu F$, $R_c = 20m\Omega$. Figure 4 shows the stability region for different cases with a fixed amplitude of the compensating ramp of $v_{ramp} = 1V$. Figure 4a shows the stability region for a specific case with a duty cycle of $d = 0.2$ (output voltage of $v_o = 5V$). The green line is the stability border numerically computed for the converter operated with the maximum output current of 0.3A. The region inside the green line is the stable region. Similarly, the red line is the stability border for the converter operated at no load. Consequently, the stability region for the converter over the whole operation is the intersection between the two areas. This is shown in figure 4a as the blue shaded area. The dotted points indicate the stability border predicted by the proposed stability criterion. As seen, it matches very well the accurate numerical stability analysis.

Figure 4b shows the stability region for a specific case with a duty cycle of $d = 0.6$ (output voltage of $v_o = 10V$). As it can be seen, the area of the stability region decreases, resulting in a complicated design of the control. Consequently, the design of the power stage has a critical role on the design of the control.

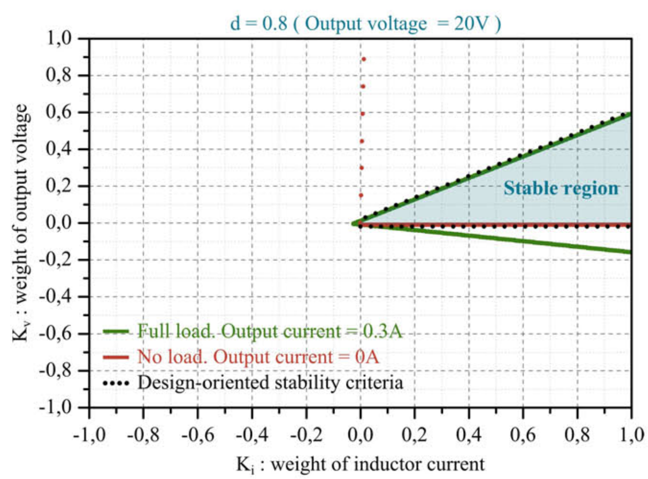
Figure 4c shows the stability region for a specific case with a duty cycle of $d = 0.8$ (output voltage of $v_o = 20V$). Notice that, now, the stability region increases.



(a) Duty cycle of 20%.



(b) Duty cycle of 60%.



(c) Duty cycle of 80%.

Fig. 4: Stable region diagrams for different duty cycles varying the weight of the inductor current and the weight of the output voltage.

IV. Dynamic response of converter

The design guidelines given in section II take into account only the stability. An additional design criterion can be derived from the point of view of the dynamic response.

When a load transient occurs in the converter, the fast loop reacts but, since the inner control does not have an integral action, a certain voltage deviation, $\triangle v_o$, occurs. This deviation is proportional to the amplitude of the output current, $\triangle i_o$ and to the dc-part of the closed-loop output impedance, $Z_{out}^{cl}(wj \to 0)$:

$$\triangle v_0 = \triangle i_o \cdot Z_{out}^{cl}(wj \to 0) \tag{14}$$

[10] shows that the dc-part of the closed-loop output impedance of a Buck converter controlled with a v^2 control compensated with inductor current is approximately equal to the current gain divided by the voltage gain. The same can be derived for the Boost converter:

$$Z_{out}^{cl}(wj \to 0) = \frac{K_i}{K_v} \tag{15}$$

This equation does not take into account the compensating ramp but it is sufficiently accurate for most cases. Then, the voltage deviation is:

$$\triangle v_0 = \triangle i_o \frac{K_i}{K_v} \tag{16}$$

Equation (16) is then an additional design criterion that takes into account the desired dynamic response.

Figure 5 shows the closed-loop output impedance of the v^2 control compensated with inductor current for a specific case with the same parameters as in section III and $v_o = 8.5V$. The voltage and current gains of the feedback path are designed according to the proposed design criteria resulting in $K_v = 0.1$ and $K_i = 0.4$.

The predicted dc-part of the closed-loop output impedance is 4.2Ω while the simulated value is 4Ω, resulting in a good agreement.

Figure 6 shows the dynamic response of the converter under a load transient $320mA \to 420mA$ without the outer slow loop. The output voltage

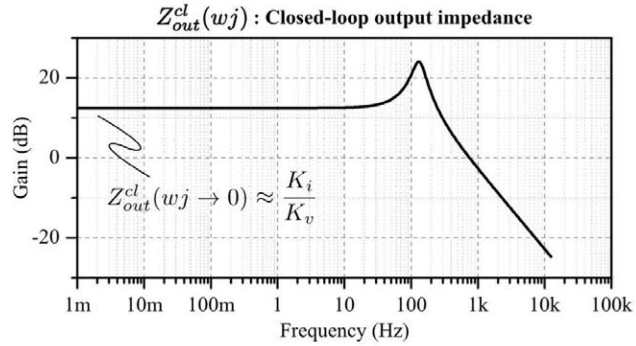


Fig. 5: Closed-loop output impedance of v^2 control compensated with inductor current.

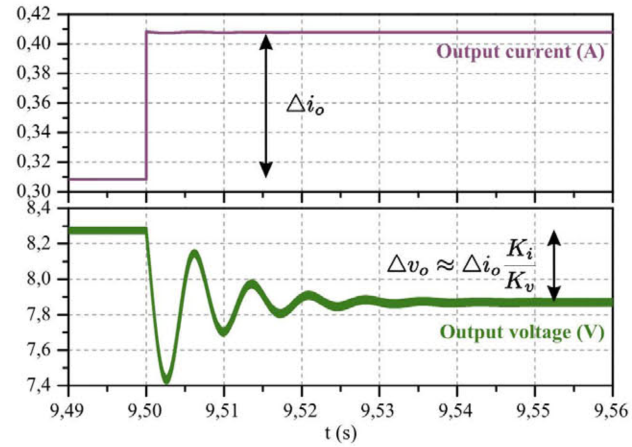


Fig. 6: Load transient $320mA \to 420mA$ of the v^2 compensated with inductor current without the outer slow loop.

oscillates because the control does not manage to attenuate the peaking in the output impedance and finally settles with an output voltage deviation of approximately $0.4V$. This is the same value as the predicted one using (16).

V. Conclusions

A design-stability criterion has been proposed for a boost converter with v^2 control compensated with inductor current. It has been validated for different duty cycles at simulation level with a very accurate numerical stability analysis. It has been shown that the design of the power stage, specially the duty cycle ratio, have a critical role on the design of the control. Specifically, duty cycles near 50% decrease the stability region and that affects the difficulty to design the control and its robustness. Also, an important conclusion is that it has been shown that it is needed to add current information in the v^2 control in order to enhance stability of the boost converter.

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